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MAXIMUM LIKELIHOOD ESTIMATION OF A DISTRIBUTION FUNCTION WITH MONOTONE FAILURE RATE BASED ON CENSORED OBSERVATIONS*

by

W. J. Padgett and L. J. Wei
University of South Carolina
Statistics Technical Report No. 39
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SUMMARY

The maximum likelihood estimator of a distribution function with monotone failure rate is derived based on a set of observations subject to arbitrary right censorship. This estimator is defined everywhere on the positive real line while the Kaplan-Meier estimator may not be. The small sample properties of this estimator are indicated by results of a Monte Carlo study for the Weibull distribution.

Some key words: Life testing; Product limit estimator; Right censorship.

1. INTRODUCTION

In life testing or survival studies, the observation of the time of occurrence of a failure or death may be prevented by the occurrence of some other event, resulting in a loss of an item or individual from the study. In this type of censoring, only the time of loss can be observed when the loss occurs before the death of the item. The problem of nonparametrically estimating a survival function from such censored data has received much attention in the recent statistical literature. Breslow & Crowley (1974) and Lagakos (1979) have given excellent reviews of this subject.

Specifically, we consider the following. Let $X_1^\circ, X_2^\circ, \ldots, X_n^\circ$ be the true survival times of n items or individuals which are censored from the right by a sequence $U_1, U_2, \ldots, U_n$ which may be either constants or random variables. It is assumed that the X_i° are independent, identically distributed random variables with a common unknown distribution function $F(t)$. We wish to estimate the survival function $\bar{F}(t) = 1 - F(t) = \Pr\{X^\circ > t\}$ based on observations consisting of a sequence of pairs (X_i, δ_i), where

$$X_i = \min\{X_i^\circ, U_i\} \text{ and } \delta_i = \begin{cases} 1 \text{ if } X_i^\circ \le U_i \\ 0 \text{ if } X_i^\circ > U_i, \end{cases} \quad i = 1, 2, \ldots, n.$$

Thus, it is known which observations are times of deaths and which ones are times of losses (censored).

One of the most popular estimators of $\bar{F}(t)$ is the product limit estimator $\hat{P}(t)$ proposed by Kaplan & Meier (1958), which was shown to be self-consistent by Efron (1967). Breslow & Crowley (1974) gave a rigorous derivation of the large sample properties of the product limit estimator. Another method of deriving the product limit estimate in a maximum likelihood framework considered by Nelson (1969) and Breslow (1972, 1974) was to restrict the set of distribution functions to those having a hazard function which was constant between the distinct uncensored failure times. The product limit estimate is a step function and is not well-defined when the largest observation is a loss. To improve on this situation, Susarla & Van Ryzin (1976, 1978) and Ferguson & Phadia (1979) proposed nonparametric Bayes estimators which are defined everywhere, use all of the censored and uncensored observations, and result in smoother estimates than the product limit estimate. Susarla & Van Ryzin's (1976) estimator still has jumps at the uncensored failure times, however.

Cox (1972, 1975) proposed the proportional hazard model with covariate variables and used the partial likelihood principle to analyze survival data.

In many situations, the life distribution $F(t)$ may be assumed or known to have a monotone hazard or failure rate function (Barlow & Proschan (1975)). For uncensored data, Grenander (1956) derived maximum likelihood estimators for the failure rate function $r(t)$ and $F(t)$ assuming only that $r(t)$ is increasing. (Throughout the paper we write "increasing" for "nondecreasing".) The consistency and asymptotic distribution of this maximum likelihood estimator were established by Marshall & Proschan (1965) and Prakasa Rao (1970), respectively.

In this paper we obtain the maximum likelihood estimator $\hat{\bar{F}}(t)$ of $\bar{F}(t)$ under the condition that $F(t)$ has a monotone failure rate based on a set of observations subject to arbitrary right censorship as described previously. This maximum likelihood estimator is continuous and well-defined for all $t \geq 0$. In Section 3 an example is presented, and in Section 4 an indication of the small sample properties of $\hat{\bar{F}}(t)$ as compared with $\hat{P}(t)$ is given, resulting from a Monte Carlo study for several Weibull distributions with increasing failure rate functions. The estimator $\hat{\bar{F}}(t)$ performs very well in the tails of the distribution and for samples under rather severe censorship.

2. THE MAXIMUM LIKELIHOOD ESTIMATOR

Let (X_i, δ_i), $i = 1, \ldots, n$ denote the sample described in Section 1, and let f denote the common probability density function of the X_i°. Assume that $U_1, \ldots, U_n$ are either constants or independent random variables which are also independent of $X_1^{\circ}, \ldots, X_n^{\circ}$. Then the likelihood function can be written as (Lagakos (1979))

$$L = L\{(x_i, \delta_i): i = 1, \ldots, n\} = \prod_{i=1}^{n} \{f(x_i)\}^{\delta_i} \{\bar{F}(x_i)\}^{1-\delta_i}.$$

Let the failure rate function be $r(t) = f(t)/\bar{F}(t)$. It follows that the likelihood function is

$$L = \prod_{i=1}^{n} \{r(x_i)\}^{\delta_i} \bar{F}(x_i), \tag{2.1}$$

and since

$$\bar{F}(t) = \exp\{-\int_0^t r(u)du\}, \; t \geq 0, \tag{2.2}$$

we can write from (2.1)

$$\ln L = \sum_{i=1}^{n} \delta_i \ln r(x_i) - \sum_{i=1}^{n} \int_0^{x_i} r(u)du. \tag{2.3}$$

Without any loss of generality, assume that $x_1 \leq x_2 \leq \ldots \leq x_n$. We also assume that r(t) is increasing. Consider the failure rate function

$$r^*(t) = \begin{cases} 0, & t < x_1 \\ r(x_i), & x_i \le t < x_{i+1},\ i = 1, \ldots, n-1 \\ r(x_n), & x_n \le t. \end{cases}$$

Then for each t, $r(t) \ge r^*(t)$, and from (2.3), we obtain

$$\begin{aligned} \ln L &\le \sum_{i=1}^{n} \delta_i \ln r(x_i) - \sum_{i=1}^{n} \int_0^{x_i} r^*(u)du \\ &= \sum_{i=1}^{n} \delta_i \ln r(x_i) - \sum_{i=1}^{n-1} (n-i)(x_{i+1} - x_i)r(x_i) \\ &\equiv \ln L^*. \end{aligned}$$

Denote the distinct uncensored failure times by $T_1 < T_2 < \ldots < T_k$, where k is the number of δ_i which equal one. Let λ_j denote the number of losses which occur in the interval $[T_j, T_{j+1})$, including any losses at T_j but not at T_{j+1}, $j = 0, 1, \ldots, k$, where $T_0 = 0$ and $T_{k+1} = \infty$. Let the times of the λ_j losses be denoted by $L_i^{(j)}$, $i = 1, 2, \ldots, \lambda_j$.

Since r(t) is increasing, $r(L_i^{(j)}) \ge r(T_j)$, $i = 1, \ldots, \lambda_j$ (for each λ_j that is not zero). Therefore, for each j, $0 \le j < k$, we have

$$\begin{aligned} &-\sum_{i=\lambda_0+\ldots+\lambda_{j-1}+j}^{\lambda_0+\ldots+\lambda_j+j} (n-i)(x_{i+1} - x_i)r(x_i) \\ &\quad = -\{n - (\lambda_0+\ldots+\lambda_{j-1}+j)\}(L_1^{(j)} - T_j)r(T_j) \\ &\qquad -\{n - (\lambda_0+\ldots+\lambda_{j-1}+j+1)\}(L_2^{(j)} - L_1^{(j)})r(L_1^{(j)}) \\ &\qquad \vdots \\ &\qquad -\{n - (\lambda_0+\ldots+\lambda_{j-1}+\lambda_j+j)\}(T_{j+1} - L_{\lambda_j}^{(j)})r(L_{\lambda_j}^{(j)}) \end{aligned}$$

$$\le -[(L_1^{(j)}+\dots+L_{\lambda_j}^{(j)}) + \{n-(\lambda_0+\dots+\lambda_j+j)\}T_{j+1}$$
$$-\{n - (\lambda_0+\dots+\lambda_{j-1}+j)\}T_j]\ r(T_j),$$

replacing $-r(L_i^{(0)})$ by zero, $i = 1, \dots, \lambda_0$. Hence

$$\ln L^* \le \sum_{j=1}^{k} \ln r(T_j) - \sum_{j=1}^{k} a_j\ r(T_j) = \ln L^{**}, \tag{2.4}$$

where

$$a_j = \begin{cases} L_1^{(j)}+\dots+L_{\lambda_j}^{(j)} + \{n - (\lambda_0+\dots+\lambda_j+j)\}T_{j+1} \\ \qquad -\{n - (\lambda_0+\dots+\lambda_{j-1}+j)\}T_j, \quad j = 1, \dots, k-1, \\ L_1^{(k)}+\dots+L_{\lambda_k}^{(k)} - \lambda_k T_k, \quad j = k. \end{cases}$$

Now, the problem of obtaining the maximum likelihood estimator of $r(t)$ subject to the condition that $r(t)$ is increasing is reduced to that of maximizing $\ln L^{**}$ given by (2.4) subject to the constraint $r(T_1) \le r(T_2) \le \dots \le r(T_k)$. Let $y_j = r(T_j)$, $j = 1, \dots, k$. Then we wish to obtain

$$\max \{\sum_{j=1}^{k} \ln y_j - \sum_{j=1}^{k} a_j y_j\} \tag{2.5}$$
$$\text{subject to } y_1 \le y_2 \le \dots \le y_k.$$

We note that if for some j, $a_j = 0$, then the function $G(y_1, \dots, y_k) = \sum_{j=1}^{k} (\ln y_j - a_j\ y_j)$ is not bounded. However, when $j \ne k$,

$$a_j > L_1^{(j)}+\dots+L_{\lambda_j}^{(j)}+\{n-(\lambda_0+\dots+\lambda_j+j)\}T_j$$
$$-\{n-(\lambda_0+\dots+\lambda_{j-1}+j)\}T_j$$
$$= L_1^{(j)}+\dots+L_{\lambda_j}^{(j)} - \lambda_j T_j \geq 0.$$

Therefore, only a_k can be zero and this occurs when there are no censored observations larger than T_k, the largest uncensored failure time. If $a_k = 0$, it is impossible to obtain a maximum likelihood estimator of $r(t)$ directly by solving (2.5). Consequently, we first consider the subclass F^M of distributions F with corresponding failure rate functions bounded by a constant M. Following the argument of Marshall & Proschan (1965) and utilizing the results of Barlow, Bartholomew, Bremner, & Brunk (1972, p. 44), the maximum likelihood estimator of $r(t)$ for $F \in F^M$ is given by

$$\tilde{r}_n^M(T_i) = \min [\min_{v \geq i+1} \max_{u \leq i} \{(v-u)/(r_u^{-1}+\dots+r_{v-1}^{-1})\}, M] \tag{2.6}$$

where $r_k = M$ and $r_j = a_j^{-1}$, $j = 1, 2, \dots, k-1$. Letting $M \to \infty$ in (2.6), we obtain the maximum likelihood estimator of $r(t)$ for F as

$$\hat{r}(t) = \begin{cases} 0, & t < T_1 \\ \hat{r}_n(T_i), & T_i \leq t < T_{i+1},\ i = 1, 2, \dots, k-1 \\ \hat{r}_n(T_k), & T_k \leq t \end{cases} \tag{2.7}$$

where

$$\hat{r}_n(T_i) = \min_{v \geq i+1} \max_{u \leq i} \{(v-u)/(r_u^{-1}+\dots+r_{v-1}^{-1})\},\ i = 1, \dots, k-1,$$

and $\hat{r}_n(T_k) = \infty$.

If $a_k \neq 0$, the solution of (2.5) is obtained by applying the results of Barlow, Bartholomew, Bremner, & Brunk (1972, p. 44). In this case, the maximum likelihood estimator of $r(t)$ is given by (2.7) where

$$\hat{r}_n(T_i) = \min_{v \geq i+1} \max_{u \leq i} \{(v-u)/(r_u^{-1}+\ldots+r_{v-1}^{-1})\},$$

and $r_j = a_j^{-1}$, $j = 1, 2, \ldots, k$.

In either case, the maximum likelihood estimator of $\bar{F}(t)$ is obtained from equation (2.2) as

$$\hat{\bar{F}}(t) = \exp\{-\int_0^t \hat{r}(u)du\}$$

$$= \exp[-\sum_{\{i:\ i \geq 0, T_i \leq t\}} \hat{r}_n(T_i)\{\min(t, T_{i+1}) - T_i\}], \quad t \geq 0, \qquad (2.8)$$

where $T_0 = 0$ and $T_{k+1} = \infty$. We note that this estimator of the survival function is well-defined for all $t \geq 0$, is a smooth function, and approaches zero as $t \to \infty$ (if $a_k = 0$, $\hat{\bar{F}}(t) = 0$ for $t \geq T_k$ since $\hat{r}_n(T_k) = \infty$).

Similar techniques can be applied for the case that F has decreasing failure rate.

3. AN EXAMPLE

We use the data given by Kaplan & Meier (1958, p. 464) and also used by Susarla & Van Ryzin (1976) to obtain an estimate of the survival function from the maximum likelihood procedure in Section 2. The ordered data are 0.8, 1.0^+, 2.7^+, 3.1, 5.4, 7.0^+, 9.2, 12.1^+ months, where + denotes a time of loss. In our notation $\delta_i = 1$, $i = 1, 4, 5, 7$ and $\delta_i = 0$, $i = 2, 3, 6, 8$, with $T_1 = 0.8$, $T_2 = 3.1$, $T_3 = 5.4$, and $T_4 = 9.2$. Also, $\lambda_0 = 0$, $\lambda_1 = 2$, $\lambda_2 = 0$, $\lambda_3 = \lambda_4 = 1$ with $L_1^{(1)} = 1.0$, $L_2^{(1)} = 2.7$, $L_1^{(2)} = 0$, $L_1^{(3)} = 7.0$, and

$L_1^{(4)} = 12.1$. The a_j are then $a_1 = 13.6$, $a_2 = a_3 = 9.2$, and $a_4 = 2.9$.

(Figures 1 and 2 about here)

Figure 1 shows the estimate $\hat{r}_8(t)$ of the failure rate function. We note that since a censored value was observed larger than T_4, $\hat{r}_8(t)$ is finite for all t. Figure 2 gives the estimates of the survival curve using $\hat{\bar{F}}(t)$ given by (2.8) and $\hat{P}(t)$, the product limit estimate. These results can be compared with the nonparametric Bayes estimate for the same data given by Susarla & Van Ryzin (1976, p. 900). We remark that to obtain the nonparametric Bayes estimate, the parameter α of the Dirichlet process prior must be chosen. Susarla & Van Ryzin (1976) indicate the effects of three choices of α on their estimate.

4. SMALL SAMPLE COMPARISONS

We have performed Monte Carlo simulations for the Weibull distribution in order to obtain the small sample behavior of the estimator $\hat{\bar{F}}(t)$ as compared with the product limit estimator $\hat{P}(t)$. The simulations were based on 2000 random samples each of size n, $\{X_i^\circ\}$, from a Weibull distribution with survival function $\bar{F}(t) = \exp(-t^\alpha/\beta)$, $t \geq 0$, with right censorship. The censoring random variables $U_1, \ldots, U_n$ were chosen to be independent of the X_i° and independent, identically distributed as uniform on $(0, T_\zeta)$, where T_ζ was the ζth percentile of the Weibull distribution. For example, when $\zeta = 75$, we obtain on the average 25% censoring in the samples (censoring fraction 0.25).

The average squared error of the estimates was computed from the 2000 trials for various values of $\alpha \geq 1$, n, and censoring fractions. Table 1 shows

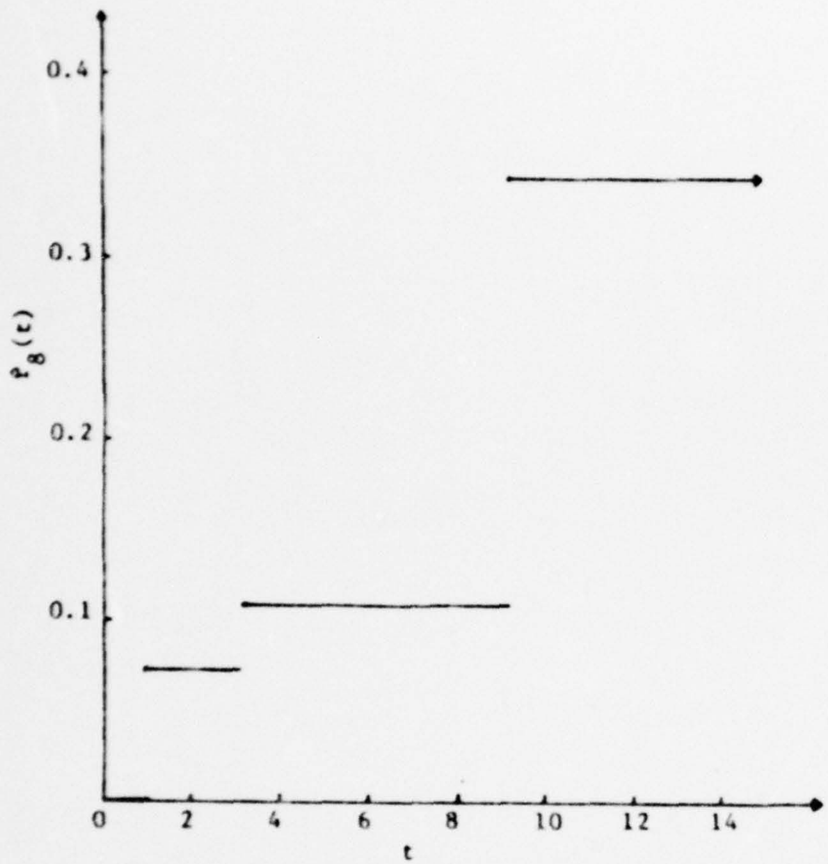

Figure 1. MLE of r(t).

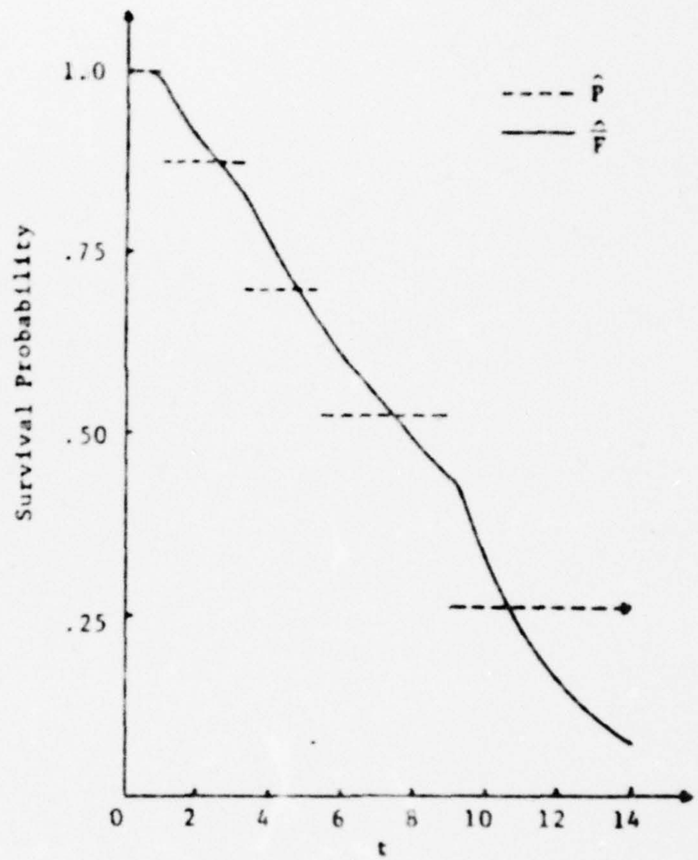

Figure 2. Estimates of Survival Probability.

Table 1. Monte Carlo Results for 2000 Trials with Weibull Distribution

Censoring Fraction			0.25				0.50				0.75			
Sample Size n			10		20		10		20		10		20	
(α,β)	t	$\bar{F}(t)$	ASE $\hat{\bar{F}}$	ASE $\hat{p}$	ASE $\hat{\bar{F}}$	ASE $\hat{p}$	ASE $\hat{\bar{F}}$	ASE $\hat{p}$	ASE $\hat{\bar{F}}$	ASE $\hat{p}$	ASE $\hat{\bar{F}}$	ASE $\hat{p}$	ASE $\hat{\bar{F}}$	ASE $\hat{p}$
(1,2)	0.21	0.9	0.006	0.009	0.004	0.004	0.006	0.010	0.004	0.005	0.007	0.012	0.004	0.005
	0.71	0.7	0.025	0.025	0.013	0.012	0.035	0.033	0.015	0.016	0.127	0.053	0.097	0.033
	1.39	0.5	0.045	0.039	0.019	0.019	0.100	0.069	0.072	0.045	0.152	0.125	0.111	0.103
	2.41	0.3	0.050	0.055	0.033	0.031	0.072	0.128	0.050	0.096	0.163	0.277	0.082	0.252
	4.61	0.1	0.014	0.103	0.008	0.071	0.044	0.268	0.012	0.227	0.205	0.509	0.065	0.481
(1.5,2)	0.35	0.9	0.006	0.010	0.004	0.005	0.007	0.011	0.004	0.005	0.011	0.015	0.005	0.007
	0.80	0.7	0.030	0.031	0.014	0.015	0.050	0.043	0.019	0.021	0.131	0.063	0.101	0.042
	1.24	0.5	0.057	0.051	0.025	0.025	0.110	0.084	0.077	0.056	0.158	0.139	0.111	0.115
	1.80	0.3	0.056	0.071	0.039	0.040	0.090	0.152	0.055	0.115	0.197	0.295	0.110	0.267
	2.77	0.1	0.024	0.127	0.011	0.084	0.085	0.299	0.029	0.254	0.278	0.530	0.129	0.500
(3.5,2)	0.64	0.9	0.010	0.017	0.005	0.007	0.015	0.020	0.006	0.008	0.036	0.029	0.011	0.013
	0.91	0.7	0.064	0.056	0.025	0.027	0.099	0.076	0.048	0.043	0.144	0.084	0.122	0.067
	1.10	0.5	0.105	0.092	0.055	0.052	0.149	0.129	0.105	0.087	0.194	0.173	0.143	0.142
	1.29	0.3	0.099	0.126	0.060	0.077	0.169	0.218	0.095	0.160	0.295	0.342	0.194	0.297
	1.55	0.1	0.098	0.211	0.037	0.139	0.233	0.388	0.109	0.313	0.459	0.591	0.293	0.532

(Table 1 about here)

some of the results. As anticipated, the performance of $\hat{\bar{F}}$ improves as the censoring becomes more severe, as α increases, and as n increases. Compared with $\hat{P}$, $\hat{\bar{F}}$ does much better in both tails of the distribution, and performs as well as $\hat{P}$ in the center based on the mean squared error. This is not surprising for the upper tail, however, since $\hat{P}$ is not well-defined when the largest observation is a loss. In the simulations, we defined $\hat{P}(t)$ to be the constant $\hat{P}(T_k)$ for $t > T_k$. Results similar to Table 1 were also found when uncensored samples were used; that is, the estimator of Grenander (1956) and Marshall & Proschan (1965) behaves in the same manner as indicated by Table 1 when compared with the product limit estimator (which is one minus the empirical distribution function for a complete sample).

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The maximum likelihood estimator of a distribution function with monotone failure rate is derived based on a set of observations subject to arbitrary right censorship. This estimator is defined everywhere on the positive real line while the Kaplan-Meier estimator may not be. The small sample properties of this estimator are indicated by results of a Monte Carlo study for the Weibull distribution.

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obtained for triangular arrays of random elements. Finally, the direct applications of these results in obtaining consistency of the kerney density estimates are indicated.